

# Visual comparison of Moving Window Kriging Models

Urška Demšar and Paul Harris

**Abstract**— Kriging is a spatial prediction method, which can predict at any location and return a measure of prediction confidence (the kriging standard error). There exist many variations of kriging, some of which can be complex, especially those that allow many of its parameters to vary spatially. To calibrate such a kriging model and to be able to interpret its results can therefore be quite daunting. We suggest that visual analytics can help with this task. In particular, we focus on the Moving Window Kriging model and three robust variants and use Star Icons to evaluate model performance and to investigate the appropriateness of the criterion used when choosing a robust model fit.

**Index Terms**—Spatial prediction, spatial statistics, kriging, geovisual analytics, multivariate visualisation, star icon maps.

## INTRODUCTION

The calibration and interpretation of a spatial statistical model is often fraught with difficulties. This can be especially true when models are specified with spatially varying parameters. We use visual analytics to help calibrate and interpret such a nonstationary model - we investigate moving window kriging (MWK) [5] together with its three robust variants [8]. Unlike any standard kriging model, the spatial autocorrelation parameters (via the variogram) of MWK vary across space, which can improve model accuracy when modelling a heterogenic process. The robust variants are of value when data are strewn with local anomalies. These models should be viewed as MWK models in development - we show that visual analytics can help furnish this development.

Previously, bivariate relationships between this study's MWK outputs have been visually explored using comaps [8]. We extend this exploration by introducing star icon maps and plots to investigate the outputs of a single MWK model and to compare several MWK models with each other. We have applied visual analytics in similar context before [3,4] and here we demonstrate its potential for the development of kriging models.

## 1 MOVING WINDOW KRIGING

Kriging is a spatial prediction technique where predictions can be found at any location together with an estimate of its uncertainty (the kriging standard error). There are numerous kriging forms [1], but here we only investigate the MWK model. For MWK, a local variogram is determined at every target location in contrast with standard forms of kriging, where there is only one global variogram produced for the entire data set. The local variogram parameters are then used to calibrate a kriging algorithm in order to provide a prediction and its standard error. The MWK models of this study can only be briefly described here - for full details see [8].

Our case study data is a freshwater acidification critical load data set covering the whole of Great Britain [2]. The size and scale of this data suits nonstationary modelling, where previous investigations have provided evidence of variogram nonstationarity [8,9], together with evidence of local anomalies [7,8]. The full data set consists of 686 critical load values, split into model calibration and validation subsets of 497 and 189 values, respectively. We used Voronoi polygons of the validation locations for visualisation purposes.

**MW-SK:** In this basic, non-robust MWK model, simple kriging (SK) is applied within some window using local variogram parameters found at the same spatial scale. At every target location, a classic variogram is first estimated and then modelled to get a smooth representation of spatial autocorrelation. Technical details are in [8]. When there are outliers in the data together with evidence of data non-normality, such a basic MWK model often produces unreliable results. This can be prevented by using a robust version - one resistant to outliers and data skewness. In the case of MW-SK,

we adapt this basic model to a robust form in three different and experimental ways.

**MW-SK-ROB-1:** MW-SK is adapted sequentially: (a) globally Box-Cox transform the data; (b) locally estimate and model robust variograms using the transformed data; (c) locally apply a robust form of SK, where the robust variogram model parameters are used to find winsorized local calibration data subsets; (d) back-transform the robust SK results to the original data space. Here each adaptation is robust and adaptation (c) edits outlying observations in order to reduce their influence on the model output.

**MW-SK-ROB-2:** Our second robust model is the same as our first one, except that the Box-Cox transform is locally specified.

**MW-SK-ROB-3:** Our third robust model is a hybrid; where at some locations MW-SK model is applied, whilst at others, ROB-MW-SK-2 is applied. For this model we calculate a local diagnostic to indicate the presence of aspatial outliers (and non-normality) in the local calibration data subset. This local robust fit criterion is evaluated through visual analysis. The procedure resulted in 57 of the 189 validation sites adopting a robust SK prediction.

## 2 VISUALISING MODEL RESULTS

Table 1 shows a selection of visually explored model outputs.

Table 1. Model outputs being visualised.

MWK model	Attribute	Explanation
MW-SK	MWARKSE	Absolute difference between
ROB-MW-SK-1	ROB1ARKSE	the absolute residual (AR)
ROB-MW-SK-2	ROB2ARKSE	and the kriging standard error
ROB-MW-SK-3	ROB3ARKSE	(KSE) of each method.
ROB-MW-SK-2	ROB2BCPos	Box-Cox parameter
		(constrained to be positive)
ROB-MW-SK-2	ROB2FrWinEd	Number of winsorised data
		edits (spatial outliers)
ROB-MW-SK-3	ROB3NoOutl	Number of aspatial outliers

The AR-KSE differences give an indication of how a kriging model performs. The AR is the actual error while the KSE is the estimated error and so the more similar the two, the better the kriging model performs. The ROB2FrWinEd and ROB3NoOutl data provide an estimate of how many outliers are present within a window centred at a given location, for spatial and aspatial outliers respectively. ROB2BCPos is an indicator of local data skewness: the closer to 0, the more non-normal the local distribution.

We used Star Icon Maps and Plots from Geoviz Toolkit [6,10] in order to visually compare model results at each validation location. Each star icon has the same number of axes emanating from the centre as there are attributes (three or four). On each of the axes, the contribution of the corresponding attribute is plotted - our attributes are all non-negative, so the value in the centre of the icon is 0 and the values at the circumference of the icon the maximum values of each respective attribute. We present two examples: (1) we compare

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performance of all four models and (2) we try to find visual evidence to support our choice of criterion to locally apply a robust fit in MW-SK-ROB-3

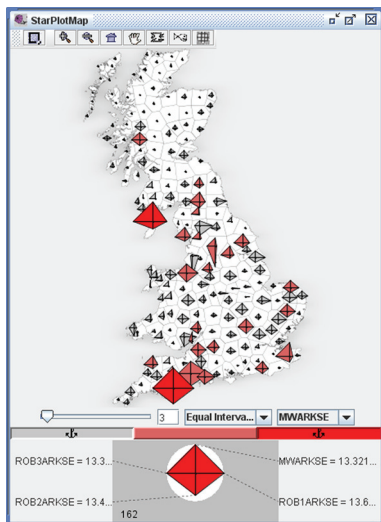


Fig. 1. Star Icon Map showing the absolute difference between the absolute residual (AR) and the kriging standard error (KSE).

**Evaluating performance of the four models:** here we visualise the absolute differences between AR and KSE for each model in a star icon. Fig. 1 shows the map with one star icon per each validation polygon, the icons placed in their appropriate geographic locations. Geoviz Toolkit also provides a table of star icons ordered according to their geometric similarity, which is not shown here due to space limit. Visual exploration of the map reveals the following patterns:

Some icons are very small (Scotland). This means that there are small differences between AR and KSE for all four models and therefore all models perform relatively well. In many other locations icons are large – suggesting the models warrant further development.

Most icons are relatively symmetrical, which means that performance is similar for all four models. A few (three or four) icons have one or more spikes – these are most easily identifiable in the icon table (not shown) and can be further investigated on the map via interactive brushing – which means that one model performs differently from all others – again useful for further development.

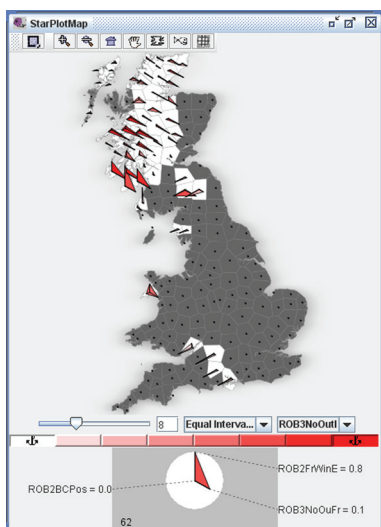


Fig. 2. Star Icons in the areas where robustness was applied locally.

**Evaluating appropriateness of the criterion for local application of a robust fit in MW-SK-ROB-3:** In the second example we investigate if the criterion for local application of the

robust fit is appropriate. We visualise the three attributes that provide indicators for presence of outliers and skewness of data: ROB2FrWinEd, ROB3NoOutl and ROB2BCPos (Fig.2). We are only interested in areas where a robust fit was actually applied in MW-SK-ROB-3 (i.e. where outliers are present and so ROB3NoOutl>0), therefore all other areas are shown with dark grey background and without icons.

Visually exploring this map in combination with the respective icon table we can identify four icon types, out of which three confirm the effectiveness of the criterion and one rejects it (the one with 0 on the top axis and high values on the bottom two axes, present in two locations in the north of England) – an observation to be used for further model development.

### 3 CONCLUSIONS

In this abstract we demonstrated how visual analytics can aid interpretation of outputs of four MWK models. Visually exploring data produced by such models (either results or model parameters) can help the method developer to evaluate different models versus each other, improve their calibration, and appropriately interpret the outputs. This is particularly important in nonstationary cases of kriging, which are often considerably complex, and which continue to be developed to address different data situations.

While we only used a certain type of visualisations in this abstract (Star Icon Maps and Plots), we continue this work with an investigation of other potentially useful multivariate visualisations for the purpose of evaluating and comparing such kriging models.

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